

22/8/2020

B.Sc. Part II 4<sup>th</sup> Paper

Differential Equations

(LDECC) (contd.)

Q. Solve  $(D^2 + 3D + 2)y = e^{2x} \cdot \sin x$

Soln

For CF

$$D^2 + 3D + 2 = 0 \Rightarrow (D+2)(D+1) = 0$$

$$\Rightarrow D = -1, -2.$$

$$\therefore CF = C_1 e^{-x} + C_2 e^{-2x}$$

For PI

$$PI = \frac{1}{D^2 + 3D + 2} e^{2x} \cdot \sin x$$

$$\Rightarrow PI = e^{2x} \frac{1}{(D+2) + 3(D+2) + 2} \sin x$$

$$\Rightarrow PI = e^{2x} \frac{1}{D^2 + 4D + 4 + 3D + 6 + 2} \sin x$$

$$\Rightarrow PI = e^{2x} \frac{1}{D^2 + 7D + 12} \sin x$$

$$\Rightarrow PI = e^{2x} \frac{1}{\cancel{D^2} - 1 + 7D + 12} \sin x$$

$$\Rightarrow PI = e^{2x} \frac{1}{4D+11} \cdot \sin x$$

$$\Rightarrow PI = e^{2x} \frac{1}{(4D+11)(4D-11)} \sin x$$

$$\Rightarrow PI = e^{2x} \frac{4D-11}{49D^2-121} \sin x$$

$$\Rightarrow PI = e^{2x} \frac{4D-11}{-49x^2-121} \sin x$$

$$\Rightarrow PI = e^{2x} \frac{4D-11}{-170} \sin x = \frac{-1}{170} e^{2x} (4D-11) \sin x$$

$$\Rightarrow PI = \frac{-1}{170} e^{2x} [4D(\sin x) - 11 \sin x]$$

$$= \frac{-1}{170} e^{2x} [4 \cos x - 11 \sin x]$$

$$\Rightarrow PI = \frac{e^{2x}}{170} (11 \sin x - 4 \cos x)$$

$\therefore$  complete soln is given by

$$y = CF + PI$$

$$\Rightarrow y = c_1 e^{-x} + c_2 e^{-2x} + \frac{1}{170} e^{2x} (11 \sin x - 4 \cos x)$$

□